Persistent Visibility Reconstruction

Liz Jimenez **Mentor: James Abello**

DIMACS and RISE, Rutgers University Summer 2009

Abstract

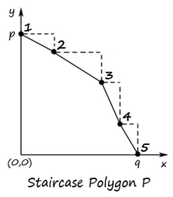
The recognition problem of visibility graphs of simple polygons has been a major open problem in Computational Geometry for over two decades. Since the visibility graph of any polygon can be dismantled as a sequence of visibility graphs of staircase polygons, a natural question is to characterize these special class. It has been shown, by J. Abello, O. Egecioglu, and K. Kumar, that visibility graphs of staircase polygons are persistent graphs. In this poster, we describe our current progress on the reverse direction, i.e. is every persistent graph the visibility graph of a staircase polygon? Our method relies on the fact that every persistent graph is the conjunction of special persistent factor subgraphs. We have devised an efficient algorithm that solves the recognition and reconstruction problem for any persistent factor. Extension of this work to the conjunction of persistent factors will furnish a solution to the general persistent graph reconstruction problem.

Staircase Polygons

Definition 1. A "staircase looking path" that connects a point p on the positive y-axis to a point q on the positive x-axis is called a **staircase path**.

Definition 2. A **staircase polygon** is a staircase path together with the segments from the origin to p and the origin to q.

Example.



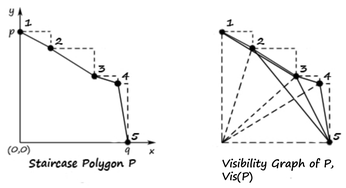
Staircase Polygon P

Visibility Graphs

Definition 3. Two vertices of a simple polygon P are **internally visible** if the open line segment between them is either $\in Boundary(P)$ or lies entirely in the interior of P.

Definition 4. The **visibility graph**, $Vis(P)$, of a simple polygon P is the graph with $V(Vis(P)) = Vertices(P)$ and whose edges correspond to internally visible pairs of vertices in P.

Example.



Staircase Polygon P

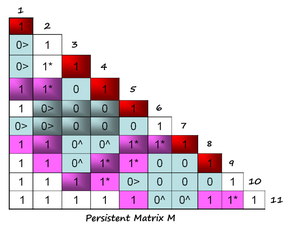
Visibility Graph of P, Vis(P)

Persistent Graphs

Definition 5. A matrix M is **persistent** if, for any $k < j < i$ such that $M(i,j) = 1$ and $M(i,y) = 0$ for every $y, k < y < j$, the following holds:

1. $M(x,y) = 0$ for $j < x < i, k < y < j$.
2. $M(j,k) = 1$ if $M(i,k) = 1$.

Example.



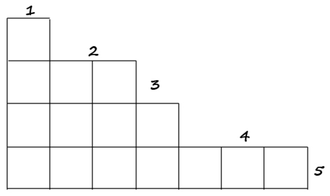
Persistent Matrix M

Theorem 6 (AEK,1995)**.** *If P is a staircase polygon, then $Vis(P)$ is a persistent graph.*

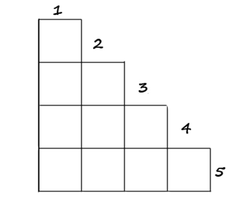
Ferrer's Diagram

Definition 7. Let $\lambda = (\lambda_1 \geq \lambda_2 \geq ... \geq \lambda_k > 0)$ be a partition of N, where N is the number of boxes. The **Ferrer's Diagram** of λ is a figure obtained from k left justified columns of squares of lengths $\lambda_1, \lambda_2, ..., \lambda_k$.

Example.



Example of Ferrer's Diagram



Special Case of Ferrer's Diagram
Staircase Shape

Balanced Tableaux

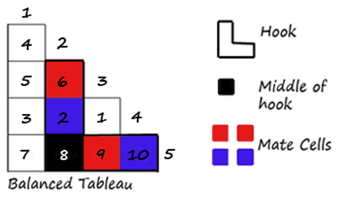
Definition 8. A **tableau** T is a Ferrer's diagram of staircase shape where each cell contains a distinct value between 1 and $\binom{n}{2}$.

Definition 9. A pair of cells $T(i,j)$ and $T(j,k), k < j < i$ are called **mates** with respect to $T(i,k)$.

Definition 10. The hook belonging to $T(i,j)$ is the collection of cells above it and to its right.

Definition 11. A tableau is **balanced** if the value in each entry is between the value of mates cells in its hook.

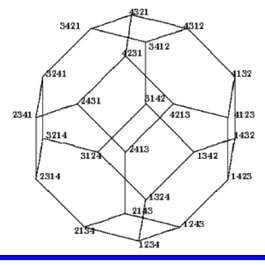
Example.



Balanced Tableau

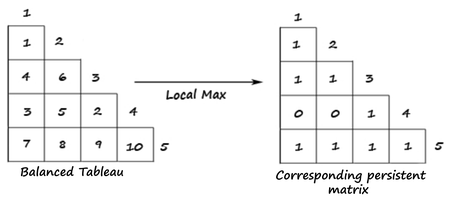
Weak Bruhat Order

Balanced tableaux $\leftrightarrow$ maximal chains in the Weak Bruhat Order of the Symmetric Group S_n.



Balanced Tableaux to Persistent Graphs

Theorem 12 (AEK,1995)**.** *Persistent graphs are obtained precisely from balanced tableaux via local maximums (or dually by local minimums).*



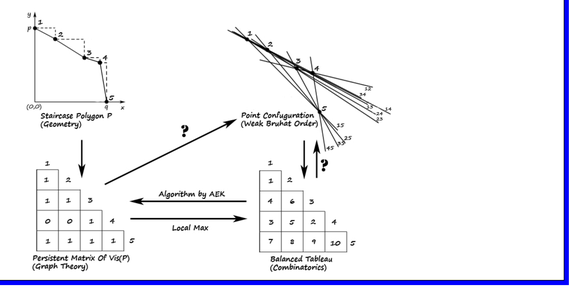
Balanced Tableau

Corresponding persistent matrix

Our Project

The question: Is every persistent graph the visibility graph of a staircase polygon?

If true, then: Those graphs obtained from balanced tableaux via local max are precisely visibility graphs of staircase polygons.



Main Ideas

1. Persistent graphs can be decomposed into special subgraphs called factors.
2. Solve the reconstruction problem for factors.
3. Compose the factors solutions.

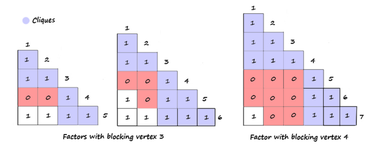
Factors

Definition 13. A vertex that blocks the visibility of two other vertices is called a **blocking vertex**.

Definition 14. A **clique** is the persistent matrix in which every entry is 1.

Definition 15. A **factor** M_i is a persistent graph with only one blocking vertex, v_i.

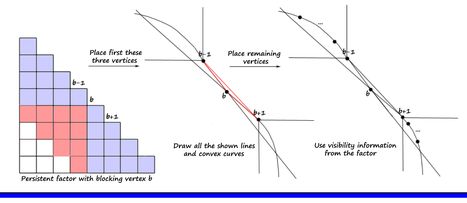
Example.



Main Result

Theorem 16 (Abello, Jimenez)**.** *If M_i is a persistent factor, then M_i is the $Vis(P)$ of a staircase polygon P.*

Reconstruction Algorithm (complexity $O(n^2)$)



Future Work

Extending this work to the general persistent graph reconstruction problem.

References

1. J. Abello, O. Egecioglu, K. Kumar, "Visibility Graphs of Staircase Polygons and the Weak Bruhat Order I: From Visibility Graphs to Maximal Chains", Discrete and Computational Geometry, Vol. 14, No 3, 1995, pp 331-358.